River basin topology, related theory of graphs

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In semi-distributed parameter modelling definition of a computational sequence is necessary

AquaLog topological algorithm uses 2 graph-based principles:

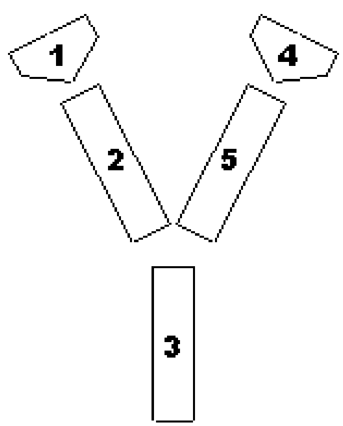
- Tree-formed hydrographic network systems are formulated using matrix of Arc-Node incidences
- Looped systems (reservoir-control systems, looped river systems) are based on the Petri Net technology

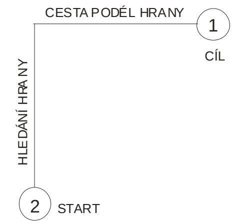
Both algorithms control the sequence of selected modelling techniques operating just as input-output transformers

Computational sequence of the basun sub-systems

Principle of the "train" :

"1"	**precedes**	"2"
"2"	**precedes**	"3"
"4"	**precedes**	"5"
"5"	**precedes**	"3"
"2"	**follows**	"1"
"3"	**follows**	"2"
"5"	**follows**	"4"
"3"	**follows**	"5"

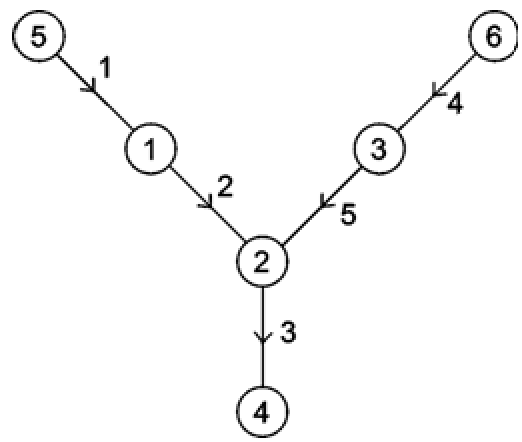




Priorities of the optimum TREE-search algorithm in matrix of incidences:

(1) The search start upwards

(2) Target of lower column number is given higher priority



MATICE INCIDENCÍ

	1	2	3	4	5	6
1	+1				1	
2	(1)	-1				
3		(1)		(-1)		
4			-1			1
5		-1	(1)			

OZNAČENÍ VRCHOLŮ

1					(5)	
2	(1)					
3		(2)		(4)		
4						(6)
5			(3)			

STROM VRCHOLŮ

1					(5)	
2	(3)					
3		(2)		(1)		
4						(6)
5			(4)			

OZNAČENÍ HRAN

1	(1)					
2		(2)				
3			(3)			
4			(4)			
5		(5)				

STROM HRAN

1	(1)					
2		(2)				
3			(1)			
4			(5)			
5		(3)				

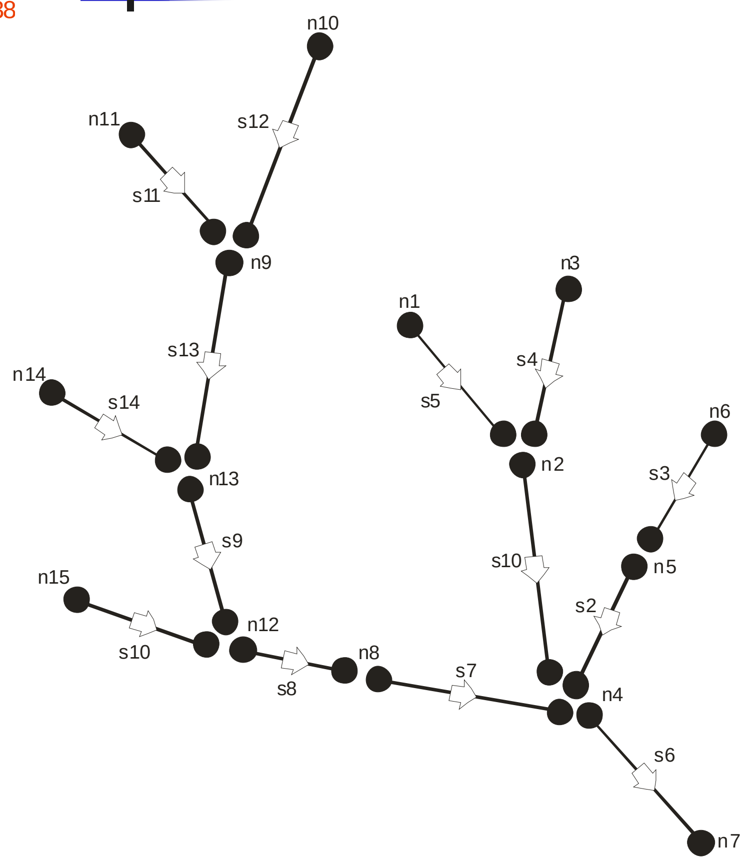
Resulting "train" :

nodes $T_n = 4, 2, 1, 3, 5, 6$

arcs $T_e = 3, 2, 5, 1, 4$

Graph of the hydrographic network and corresponding incidence matrix

3.38



			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	n																
S01	1			-1	-1												
S02	2				-1	1											
S03	3					-1	1										
S04	4			-1	1												
S05	5			-1													
S06	6				1			-1									
S07	7				-1				1								
S08	8								1								
S09	9													-1	1		
S10	10													1			-1
S11	11										-1		1				
S12	12										-1	1					
S13	13										1				-1		
S14	14														-1	1	
δ_N				3	1	4	2	1	1	2	3	1	1	3	3	1	1

Computational sequence,
"train " of semi-distributed system

$$\mathbf{T}_v = (7,4,2,5,8,3,1,6,12,13,15,9,14,11,10)$$

The graph theory
at nonstationary structured systems. Petri Nets PN

What are the Petri Nets ?

- PN consists of 4 components: Places, Transitions, Arcs, Tokens, Markings
- Two basic objects exist: state and action (change of the state)
- graph PN is defined by a pentad:

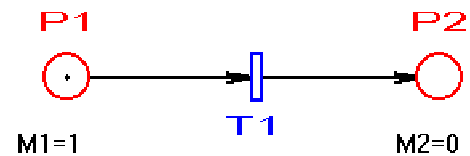
  places **P,** transitions **T**, forward and backward incidence matrix **F** and **B** and initial marking M_o

PN**=(P, T, F, B,** M_o**)**

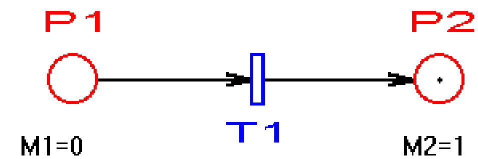
Why PNs in water management ?

- time **invariant** structure of WRS (time independent relatios between WRS components) is rather an exception (trees, cicrcles)
- Structure-formulation of the WRS objects is more likely **nonstacionary** (processes and objects in a basin and control-systems)
- changing computational environment (in)accessible observation, technological conditions etc.

PN before action („firing-up T1“):



PN after action („ firing-up T1“):

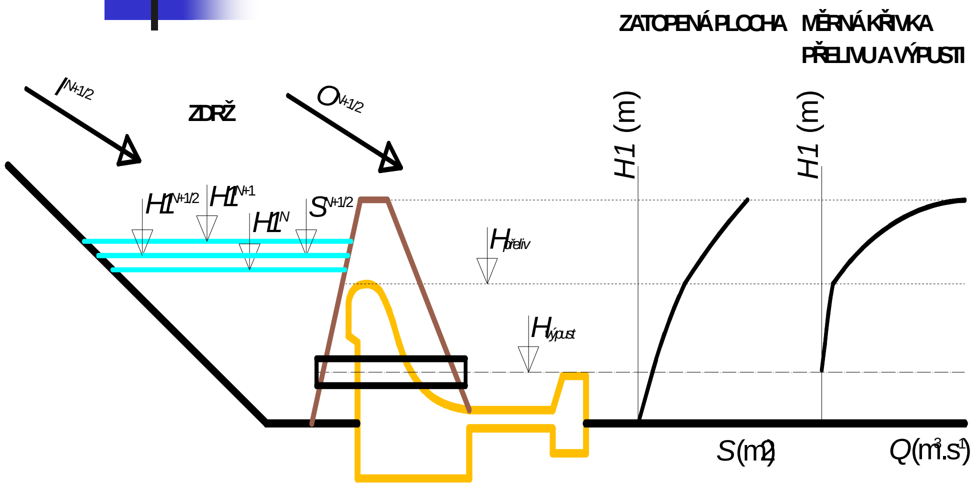


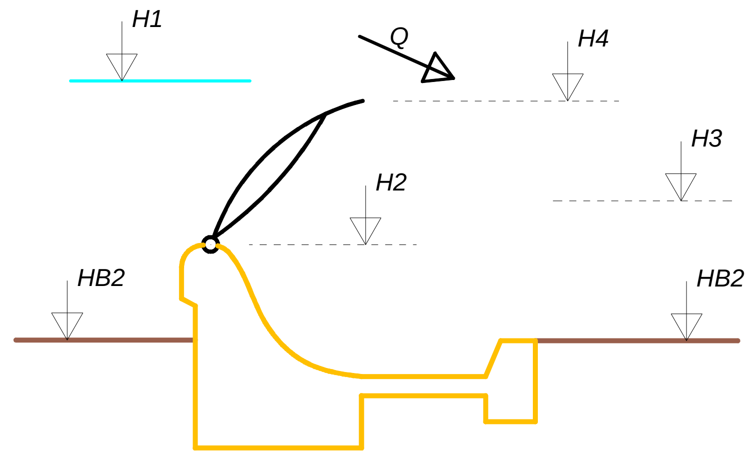
Properties:

Place	→	state of the object
Transition	→	action
Arc	→	relation P and T
Marking	→	feasibility of the Place

Elementary example of PN application: (1/3)

- connection of reservoir balance
- rating curve of the spillway

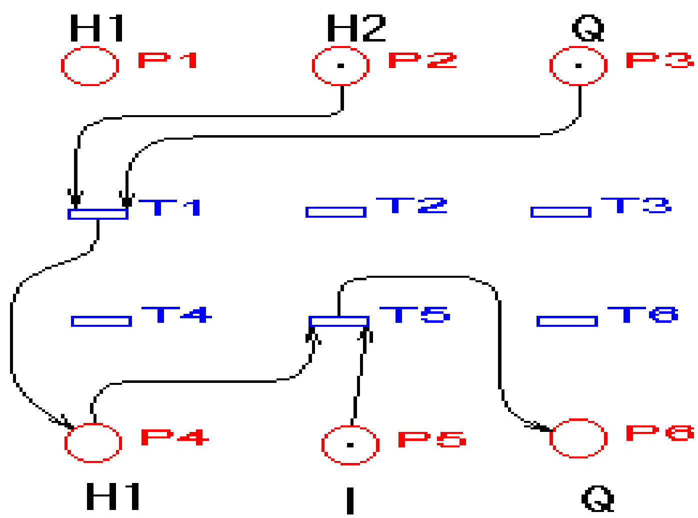




PN variants of interconnected processes:

Variants of spillway eq.

Variants of reservoir eq.



Elementary example of PN application: (2/3)

truth-table of reservoir computational variants

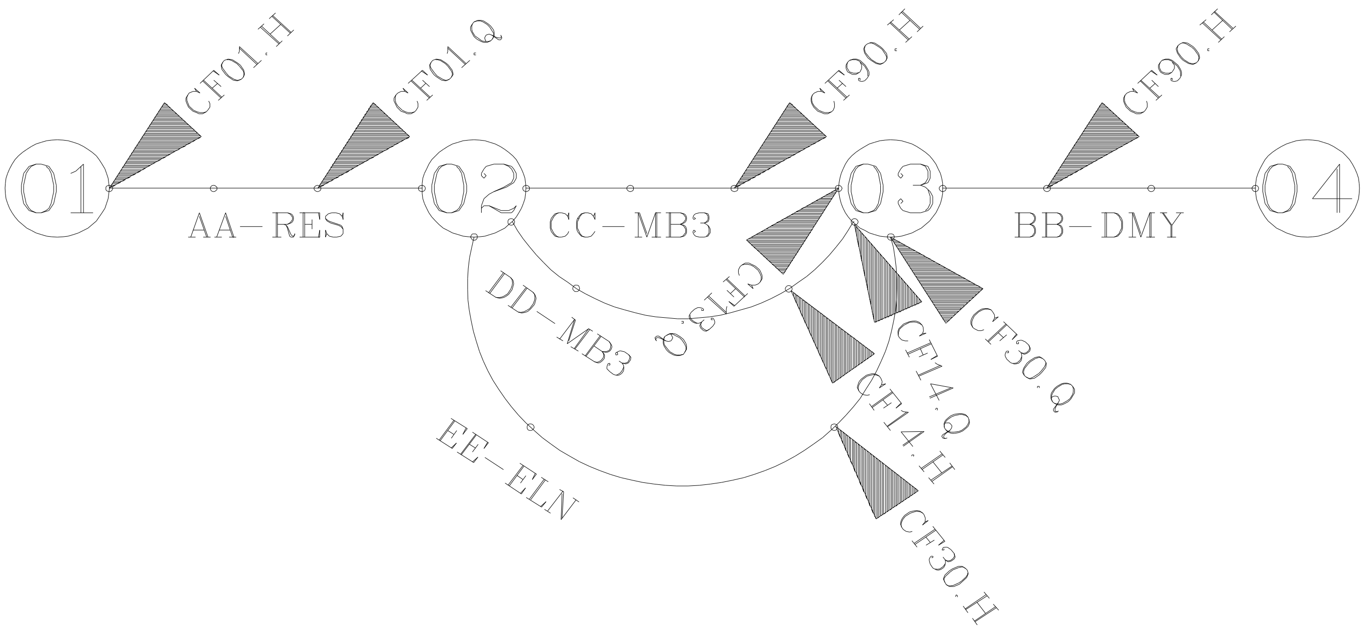
	State variables				Formulation variants		Iters
Var.	$H1^{N+1}$	$H4^{N+1}$	I^{N+1}	O^{N+1}	**step 1**	**step 2**	
1	1	1	0	0	$O^{N+1}=F1(H1^{N+1}, H4^{N+1})$	$I^{N+1}=FI(H1^{N+1}, O^{N+1})$	0
2	0	1	1	0	$O^{N+1}=F1(H1^{N}, H4^{N+1})$	$H1^{N+1}=FII(H1^{N+1}, O^{N+1})$	1
3	0	0	1	1	$H1^{N+1}=FII(H1^{N+1}, O^{N+1})$	$H4^{N+1}=F3(H1^{N+1}, O^{N+1})$	1
4	1	0	0	1	$H4^{N+1}=F3(H1^{N+1}, O^{N+1})$	$I^{N+1}=FI(H1^{N+1}, O^{N+1})$	0
5	1	0	1	0	$O^{N+1}=FIII(H1^{N+1}, P^{N+1})$	$H4^{N+1}=F3(H1^{N+1}, O^{N+1})$	0
6	0	1	0	1	$H1^{N+1}=F2(H4^{N+1}, O^{N+1})$	$I^{N+1}=FI(H1^{N+1}, O^{N+1})$	0
7	0	U	1	1	$H1^{N+1}=FII(H1^{N+1}, O^{N+1})$	*n.a.*	1
8	1	U	0	1	$I^{N+1}=FI(H1^{N+1}, O^{N+1})$	*n.a.*	0
9	1	U	1	0	$O^{N+1}=FIII(H1^{N+1}, P^{N+1})$	*n.a.*	0

Elementary example of PN application: (3/3)

truth-table of spillway computational variants

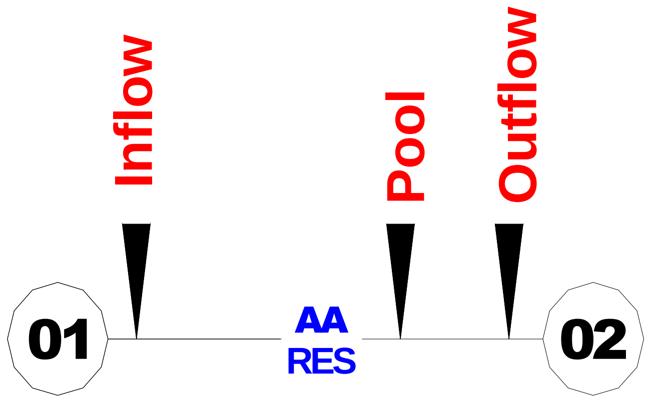
	State variables				Type of spillway	
Var..	***H1***	***H4***	***H3***	***Q***	**Sharp crested**	**Broad crested**
1	1	1	0	0	$H1\text{-}H4+k1=(Q/M1)^{2/3}$ $M1 = f(\sigma_z, m, H1, H4, H3)$	$H1+k1=H2+k2+h_{z2}=H3+k3+h_{z2}+h_{z3}$; $M2=\varphi^2 .2gB^2$ $H1=H2+Q^2/(M2(H\text{-}H4)$; *pro* $H3>H5 \Rightarrow H=H3$; *pro* $H3 \leq H5 \Rightarrow H=H2 \approx H_s$
2	1	0	1	0	$H1\text{-}H4+k1=(Q/M1)^{2/3}$ $M1=f(\sigma_z, m, H1, H4, H3)$	$H1+k1=H2+Q^2/(M2(H2\text{-}H4)^2)$ $H4^2(H1\text{-}H2)+H4(2H2^2\text{-}2H1.H2)+H1.H2^2\text{-}H2^3\text{-}Q^2/M=0$
3	1	1	0	1	*n.a.*	*for* $H3 \approx H2 \Rightarrow H1+k1=H2+Q^2/(M2(H2\text{-}H4)^2)$ $H2^3+H2^2(\text{-}H1\text{-}2H4)+H2(2H1H4+H4^2)\text{-}H1H4^2\text{-}Q^2/M2=0$
4	1	1	1	0	$Q=M1(H1\text{-}H4+k1)^{3/2}$ $M1=f(\sigma_z, m, H1, H4, H3)$	$H1+k1=H2+Q^2/(M2(H2\text{-}H4)^2)$ $Q=\sqrt{(M2)}(H2\text{-}H4)\sqrt{(H1\text{-}H2+k1)}$

Abstraction of reservoir objects and its PN

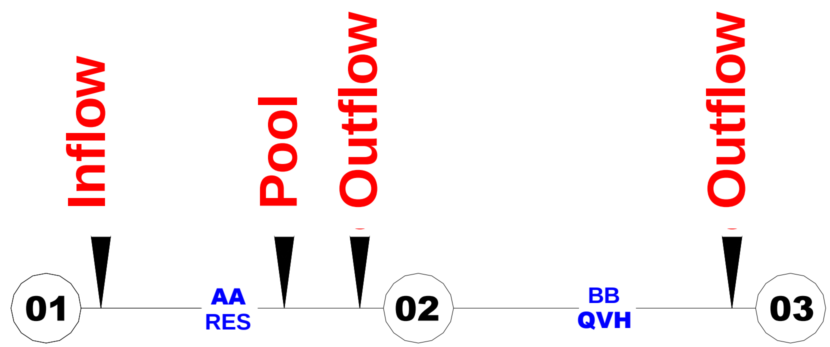


Graph of the generic reservoir

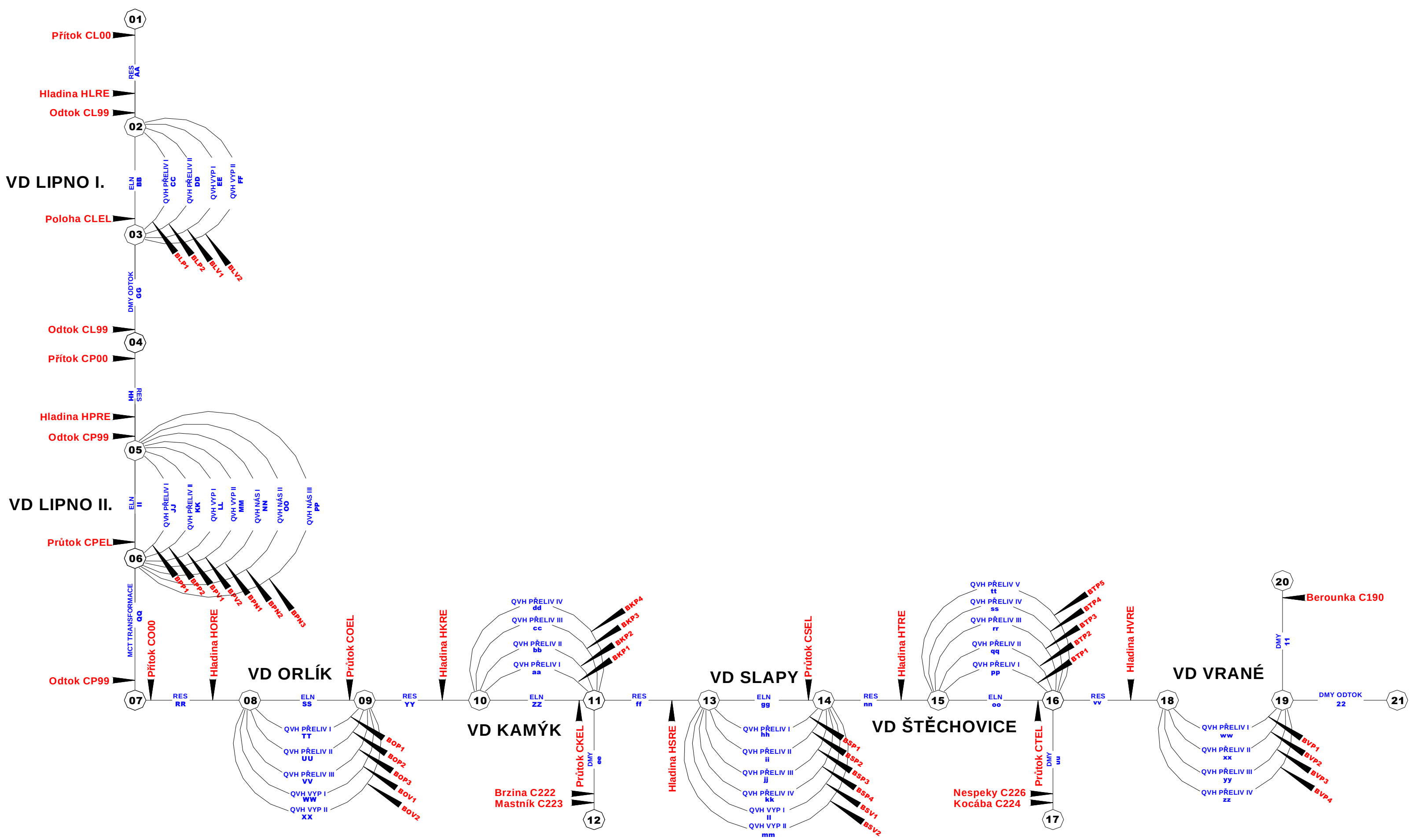
Basic:



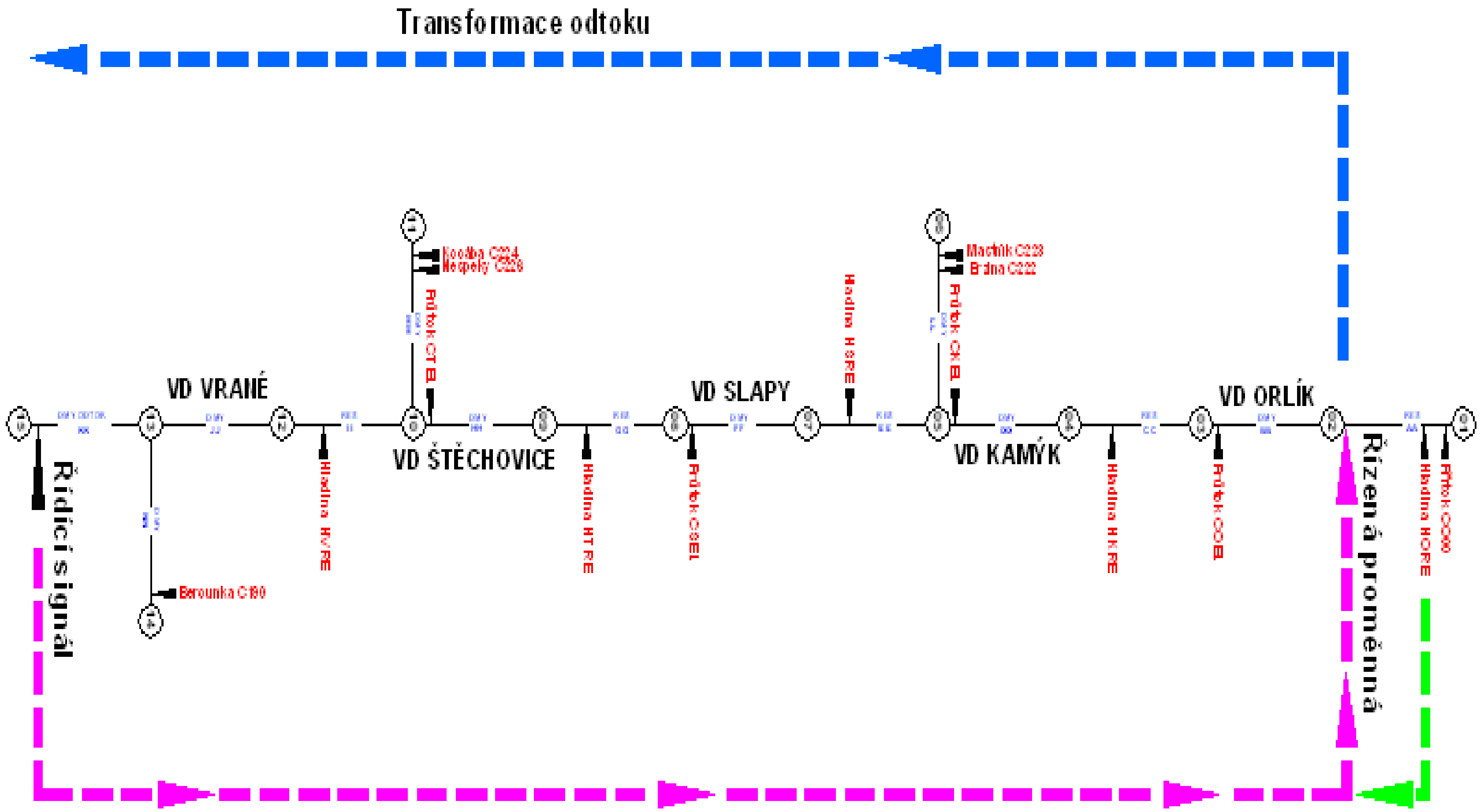
Combined with the rating curve:



PN of Vltava cascade



Vltava cascade.
Reservoirs flood control, feedback



End of 05-GRF_THEORY